Recitation 9/26 Recitation

1. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ 2 & 4 \end{bmatrix}$.

   (a) Find bases for $N(A)$ and $C(A^T)$ and sketch these subspaces together in $\mathbb{R}^2$.

   (b) Find bases for $C(A)$ and $N(A^T)$ and sketch these subspaces together in $\mathbb{R}^3$ (you can try to sketch this by hand or visit `https://www.geogebra.org/m/t4cwdjqa`).

2. Let A be an $m \times n$ matrix. What can you say about m and n if:

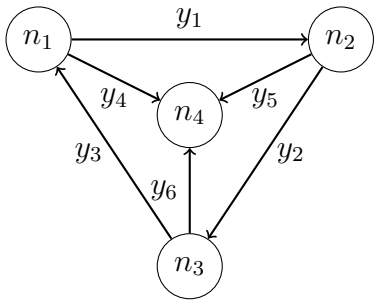
   (a) $C(A) = \mathbb{R}^m$?

   (b) $N(A) = \{\mathbf{0}\}$?

   (c) $N(A) = N(A^T) = \{\mathbf{0}\}$?

   (d) $C(A) = \mathbb{R}^m$ and $C(A^T) = \mathbb{R}^n$?

3. **Left Nullspace Application:** Consider the graph below consisting of edges and nodes.



Suppose this is an electrical network and y_i represents the current traveling along each edge as labeled above; the arrows on the edges indicate the direction of the current **Kirchhoff's Current Law** says that "current in = current out" at every node. For example, at node n_1, we have $y_3 = y_1 + y_4$. We can summarize this network using an **incidence matrix** defined as follows:

- The columns correspond to the nodes
- The rows correspond to the edges
- The entries are $a_{ij} = \begin{cases} -1 & \text{if edge } y_i \text{ leaves node } n_j \\ 1 & \text{if edge } y_i \text{ enters node } n_j \\ 0 & \text{otherwise} \end{cases}$

(a) Write down the incidence matrix for the graph above.

(b) Write the four equations for Kirchhoff's Law at the four nodes.

(c) Show that solutions to the above system correspond to solutions to $A^T\mathbf{y} = \mathbf{0}$ (i.e. the left nullspace of A).

(d) Find a basis for $N(A^T)$.

(e) Optional: Interpret each vector in the basis in terms of cycles in the network. Explain why any other cycle in the network can be built by combining cycles in this basis.